Présentation bibliographique Mensuelle aux Cordeliers

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CONTINUOUS TIME SURVIVAL IN LATENT VARIABLE MODELS

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- Models : Finite mixtures of Cox regression with and without class-specific baseline hazards, multilevel Cox regression, and multilevel frailty models.
- Joint modeling of survival time variables and continuous and categorical observed and latent variables.
- Simulation study : parameters estimation.
- Mplus Software (not free !) vs. SAS.

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- Hazard function : the event rate at time t conditional on survival until time t or later
  i.e. $h(t) = \lim_{\mathrm{d}t \to 0} \frac{\mathbb{P}[t \leq T < t + \mathrm{d}t]}{S(t)\mathrm{d}t} = \frac{f(t)}{S(t)}$

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- Parametric vs. non-parametric model for the baseline function λ.

Parametric PH Model

- $$\lambda(t) = \begin{cases} h_1 & \text{if } 0 < t \leq t_1 \\ h_2 & \text{if } t_1 < t \leq t_2 \\ \ldots & \\ h_Q & \text{if } t_{Q-1} < t \leq \infty \end{cases}$$

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- Treat h_i as regular parameters, for instance with the constraints $h_i = \alpha s(\alpha h(i - 0.5))^{s-1}$
- Treat h_i as nuisance parameters directly estimated with ML method.

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- Likelihood function that holds for both models :

$$L(T) = (\lambda(T)\exp(\beta X))^{1-\delta} S(T)$$

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- Estimator performs well on simple models : see Table 3 for application.

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- For very simple examples, parametric methods do slightly betten than non-parametric ones, only for small sample size. And t_i known a priori !

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- Again on very simple simulations ($d = 1$), Mplus method seems to better estimate β.

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- Comparison with Mplus and SAS algos (with and without ties).