Groupe de Travail des Doctorants

Modèles de survie en grande dimension

Un modèle de detection automatique de cut-points dans un modèle de Cox.

Simon Bussy

Le 11 Avril 2017

One-hot encoding and binarsity

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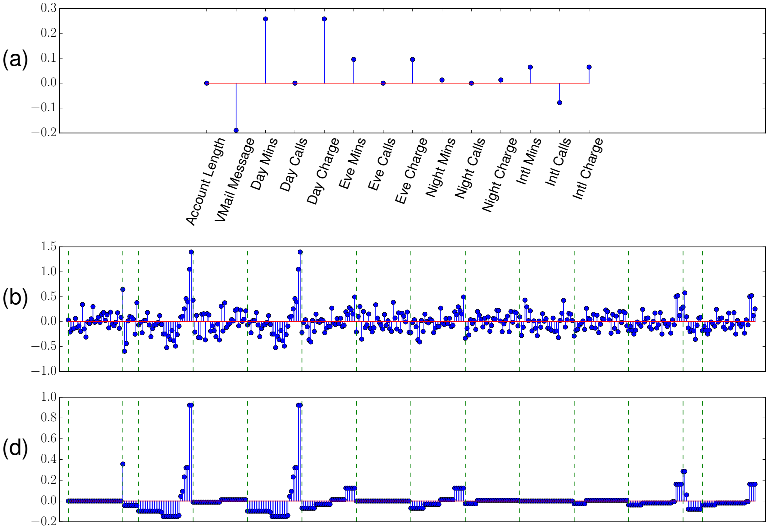
Illustration binarsity



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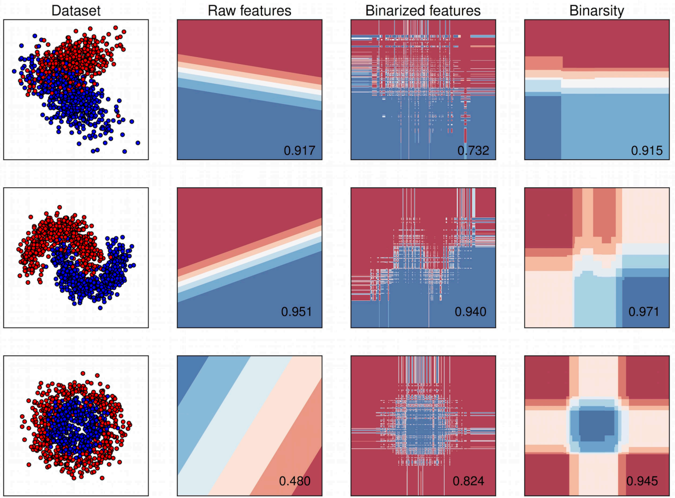
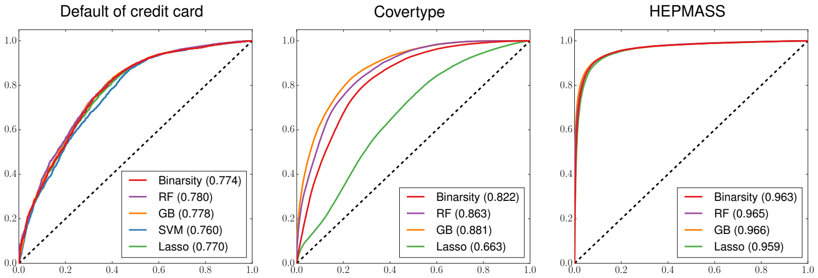
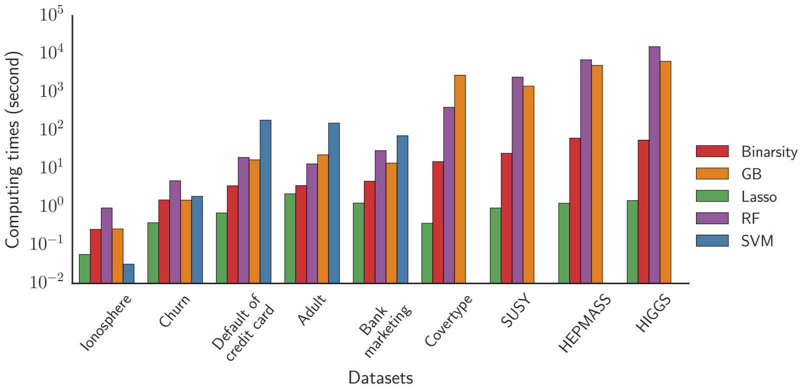


Results


Default of credit card
Binarsity (0.774)
RF (0.780)
GB (0.778)
SVM (0.760)
Lasso (0.770)
Covertype
Binarsity (0.822)
RF (0.863)
GB (0.881)
Lasso (0.663)
HEPMASS
Binarsity (0.963)
RF (0.965)
GB (0.966)
Lasso (0.959)

Computation time



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